

3-D Inversion of Airborne Electromagnetic Data

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Introduction

The 3-D inversion of airborne electromagnetic data is among the most challenging tasks in geophysics. It requires the solution of extremely large systems of linear equations arising from the discretization of Maxwell's curl-curl equations on Finite Element grids for multiple frequencies. The moving transmitter source results in a block of right-hand-side vectors which adds further to the numerical complexity of the problem. For the forward problem, we aim at the solution of families of large equation systems using efficient multigrid solvers for Maxwell problems on Nédélec type Finite Element grids. Second, numerical techniques on the basis of Gauß-Newton, Newton, or Nonlinear Conjugate Gradient type approaches will be implemented for the solution of the inverse problem. In the poster, we give an overview of the forward and inverse modelling framework and indicate the general exchange of data and results with all groups participating in the AIDA project.

Scope of the Sub-Project SP3

The sub-project aims at the development of methods for the three-dimensional inversion of helicopter electromagnetic (HEM) data. For this purpose, numerical algorithms have to be developed and implemented. These numerical algorithms utilize a recently developed fast and accurate 3-D forward operator for the reconstruction of the spatial distribution of the main petrophysical parameter, the electrical conductivity.

It is clearly not advisable to invert a complete HEM dataset. Instead, a properly chosen subset of the data is required. To this end, an indicator has to be defined which provides those parts of the dataset that cannot be interpreted using conventional 1-D inversion methods. Sub-project SP2 selects parts of the full dataset, which indicate strong lateral variations in the models obtained by a local 1-D interpretation. The reduced partial datasets, which are essentially smaller than the complete dataset, will be inverted in a subsequent step by SP3. The obtained local 3-D model will then be integrated into the quasi-1-D models of SP2. Finally, all obtained 3-D models have to be scrutinized with respect to hydrogeological and sedimentological aspects by SP5.

Milestones of SP3

- Implementation of multigrid solvers
- Implementation of efficient methods to compute sensitivities
- Tests of various regularized optimization approaches such as Gauß-Newton and Newton-like methods
- Integration with existing forward code
- Model studies
- Inversion of field data

Model studies

We note that synthetic model studies provide much insight in the nature of EM inverse problems. It must be pointed out how the low resolving power of EM methods in general and constrained inversion strategies in particular are combined to reconstructed parameter distributions, which may deviate from the original parameter distribution remarkably. This has to be communicated with all sub-projects in an early stage of the project.

The Forward Problem

The solution of the discrete forward problem of electromagnetics leads to linear equations systems with large, sparse, and structured coefficient matrices. Algebraically, the movement of the EM transmitter towed by the helicopter is expressed by multiple right-hand sides of the linear equation systems. The number of column vectors of the right-hand side block is equivalent to the number of discrete transmitter positions. We note that the coefficient matrix remains unchanged. There exist several numerical techniques to solve that kind of equation systems. Among the most important are iterative solvers, which seem attractive due to their low memory requirement when 3-D large scale problems are considered. Recently, multigrid methods become an interesting alternative to the generally slowly convergent iterative methods. We plan to solve systems with multiple right-hand sides with Krylov methods and their block-related variants.

$$\underbrace{(\nabla \times \nabla \times + i\omega\mu_0\sigma(\mathbf{r}))}_{K} \underbrace{\mathbf{E}(\mathbf{r})}_u = \underbrace{-i\omega\mu_0\mathbf{J}^e}_f$$

$$\begin{matrix} K_1 & & & & & & u_1^{1\dots N} & f_1^{1\dots N} \\ & K_2 & & & & & u_2^{1\dots N} & f_2^{1\dots N} \\ & & K_3 & & & & u_3^{1\dots N} & f_3^{1\dots N} \\ & & & K_4 & & & u_4^{1\dots N} & f_4^{1\dots N} \\ & & & & K_5 & & u_5^{1\dots N} & f_5^{1\dots N} \end{matrix} =$$

K_i complex and typically $\approx 10^6 \times 10^6$, $N \approx 1000$, $u := E, B$

The Inverse Problem

The 3-D inverse problem of HEM data interpretation is underdetermined and ill-posed. Moreover, HEM datasets are spatially incomplete and noisy. State-of-the-art inversion methods are predominantly based on regularized Gauß-Newton methods. The involved penalty functionals, which have to be minimized, typically comprise linear combinations of a data residual and a model norm. The major objective of this research is to decide which variants of known 3-D inversion methods are particularly suited for the interpretation of HEM data.

